SPECIAL SESSION ON

MODELING AND ANALYSIS OF CONTAINER TERMINAL OPERATIONS

ALLOCATING BERTHS, QUAY CRANE AND INTERNAL TRUCKS IN CONTAINER TERMINALS

Ahmed Karam

Industrial Engineering and Systems Management, Egypt-Japan University of Science and Technology, PO Box 179 New Borg Elarab City, Alexandria, Egypt. Email: ahmed.mostafa@ejust.edu.eg

ABSTRACT: This paper introduces the decisions of allocating berths, quay cranes and internal trucks in container terminals and some solution methods.

Keywords: container Terminal, operations research, berth allocation, quay crane assignment, internal trucks.

INTRODUCTION

Container terminals have become a vital connection in the global supply chain. Containerized trade grew with an average annual rate of 6.5% from 1996 to 2013(UNCTAD, 2013). With increasing number of containers, more resources such as berths, quay cranes, yard cranes and internal trucks, are necessary for container operations. However, purchasing additional resources is not a tractable decision due to the high costs of these resources. Therefore, optimizing terminal operations became necessary for container terminals, especially in such competitive environment.

The competitiveness of a marine container terminal depends on different factors, such as transshipment time combined with rates of loading and dischargingand fast turnover of containers, which corresponds to a reduction of the service time and, consequently, of the cost of the whole transportation process. A marine terminal must be managed in such a way to optimize the flow of containers that arrive and leave it in various ways, as, for instance, by trucks, trains and vessels.

A terminal can be viewed as made up of many interrelated logistic operations as shown in figure 1. The major operational planning decision problems related to these logistic operations are listed in table 1(Meisel, 2011).

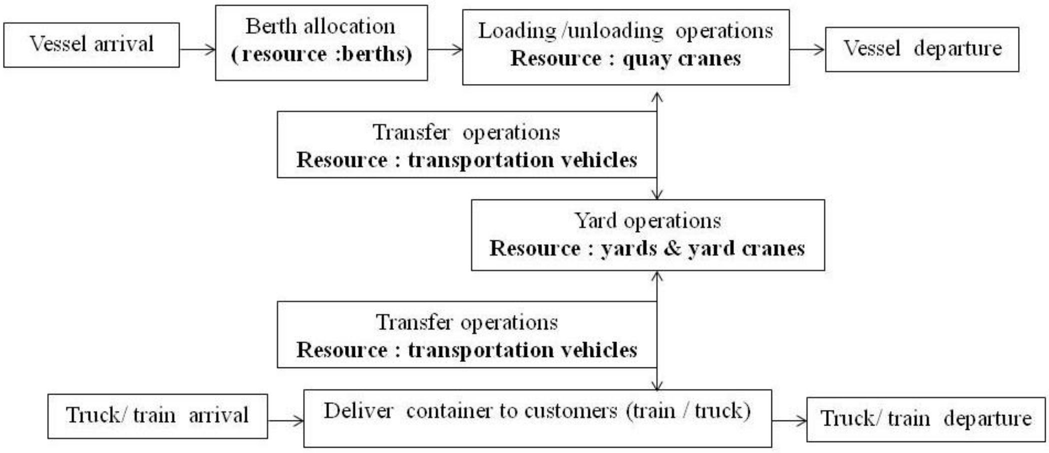


Figure (1) operations in container terminals.

By carefully analyzing the containerized trade in the Egyptian ports, it has been found that transit containers represent the majority of the total containers handled in the Egyptian ports. This is due the geographical location of Egypt that is considered in the middle of the world. Therefore, The Egyptian ports can serve for transshipment cargo bound to the North African region (Egypt, Libya, Algeria), the East Med-Levant region (Israel, Jordan, Lebanon, Syria, Cyprus, lower Turkey), Aegean region (Greece), even as far as the Adriatic (Italy, Slovenia, Croatia, Albania) and the Black Sea region. When ports handle transshipment cargo all the operations are concentrated between the quay and the yard and, in particular, congestion issues raise when mother vessels and feeders are performing simultaneously loading and unloading operations. Hence, in this paper, we focus on the quay side operations (berth allocation, quay crane assignments) and the transfer operations (assignments of transportation vehicles to quay crane).

Table (1) major operational planning descison problems of a container termianl

Planning Task	Decisions to Make
Berth allocation	Berthing positions and berthing times of vessels to be served
Quay crane assignment	Number of QCs that serve a vessel
	Specific cranes to use for the service
Quay crane scheduling	Scheduling of loading and unloading operations for a vessel
Stowage planning	Slot positions of export containers within a vessel
Yard management	Reservation of yard capacity for liner services
	Selection of storage locations for individual containers
	Remarshalling operations of containers
Yard crane scheduling	Deployment of cranes to yard blocks
	Scheduling of stackings and retrievals of containers
Horizontal transport management	Assignment of vehicles to QCs
	Scheduling of transport orders on vehicles
Hinterland operations planning	Service order of trucks and trains
	Sequencing loading and unloading operations
Workforce planning	Provision of workforce capacity
	Scheduling of labor tasks

The berth allocation problem

The berth allocation problem consists of assigning and scheduling ships to berths (discrete case) or to quay locations (continuous case) over a given time horizon. Constraints usually taken into account includes the ship's length, the berth's depth, time windows on the arrival and departure times of vessels, priority ranking, favorite berthing areas. The typical time horizon is up to one week for operational berth allocation and up to one month for tactical berth allocation.

In the following, we provide an example of an integer programming model for a discrete berth allocation which has been proposed by by Imai et al.(2001). In the problem, it is required to determine the ship-berth-order assignment with the goal of minimizing the waiting and handling time of incoming ships. All the ships are already in port when the berthing plan is determined. This guarantees every potential ship-berth-order assignment is feasible in the berth allocation. The model is developed based on the following assumptions:

(1) All the ships are already in port when the berthing plan is determined.
(2) Each berth can accommodate one ship at a time.
(3) The ship handling time is dependent on the assigned berth.
(4) There are no physical restrictions on berth, such as ship draft and water depth.

Assume that a container terminal has I berths and each berth is denoted by $i \in \{1,2,...,I\}$ and each berth can serve one ship at a time. Let S_i denote the time when berth i becomes available for the berth allocation process. There are T ships which have already arrived at the port for berthing before the berth plan is determined. For each ship $j \in \{1,2,...,T\}$, its arrival time is denoted by A_j. It is also assumed that $S_i \geq A_j$, for all $i \in \{1,2,...,I\}$, $j \in \{1,2,...,T\}$. Let C_{ij} be the container handling time spent by ship j at berth i. The problem is how to determine the optimal ship-berth-order assignment to minimize the overall waiting time plus the handling time. In the formulation, a binary variable x_{ijk}, which is the decision variable of this model, is used to indicate the service sequence of berth. It is equal to 1 if the ship j is served as the kt^h vessel at berth i . Otherwise, it is 0.

$$[\text{PS}] \qquad \text{Minimize} \quad \sum_{i\in B}\sum_{j\in V}\sum_{k\in O}\{(T-k+1)C_{ij}+S_i-A_j\}x_{ijk} \tag{1}$$

Subject to

$$\sum_{i\in B}\sum_{k\in O}x_{ijk}=1 \quad \forall j\in V, \tag{2}$$

$$\sum_{j\in V}x_{ijk}\leqslant 1 \quad \forall i\in B,\ k\in O, \tag{3}$$

$$x_{ijk}\in\{0,1\} \quad \forall i\in B,\ j\in V,\ k\in O, \tag{4}$$

where

$i\ (=1,\ldots,I)\in B$	set of berths
$j\ (=1,\ldots,T)\in V$	set of ships
$k\ (=1,\ldots,T)\in O$	set of service orders
S_i	time when berth i becomes idle for the berth allocation planning
A_j	arrival time of ship j
C_{ij}	handling time spent by ship j at berth i
x_{ijk}	1 if ship j is serviced as the kth ship at berth i
	0 otherwise

The objective (1) minimizes the sum of waiting and handling times for every ship. Constraint set (2) ensures that every ship must be serviced at some berth in any order of service. Constraint set (3) enforces that every berth services up to one ship at any time.

Assignments of quay cranes and internal trucks

In the above formulation of the berth allocation model, the handling time of the ship is assumed to be known in advance. However, a reliable estimate for handling times can be determined based on the number of quay cranes assigned to the vessel, the handling rate of the quay crane, the number of internal trucks assigned to the quay crane and the distance between the berth, where the vessel is moored, and the storage location of the containers. The assignment problem of handling resources, such as quay cranes and internal trucks, arises when several vessels are berthed simultaneously at the quay.

The berth schedule, obtained by solving the berth allocation model, is then used to assign the quay cranes and internal trucks to the incoming vessels such that the discharging/loading operations, for each vessel, are completed without violating its promised departure time. Minimizing the handling times of the vessels as well as minimizing the numbers of quay cranes and internal trucks utilized are the typical objectives of the handling plan. The number of quay cranes serving the vessel simultaneously is often restricted by minimum and maximum limits that must be respected. The handling rate of the quay crane highly depends on how many internal trucks assigned to it because there is often no buffer space below the quay crane. Thus, the continuity of the container handling by the quay crane is restricted by the availability of the internal trucks. If the internal truck is not available to pick up or deliver the container from or to the quay crane, the operation of the quay crane will be interrupted

(Chen et al., 2013). Therefore,a set of internal trucks is required to transport containers in order to keep the cranes operations uninterrupted. In practice, the number of internal trucks assigned to each quay cranes should be within a specific range which is determined by the terminal planner such that the idle times of the cranes and the internal trucks are practically acceptable.

In the following, we present an example of a MIP model for a simultaneous assignment of quay cranes and internal trucks which has been proposed by (Karam et al., 2015). The proposed model provides the number of quay cranes assigned to each vessel and it also determines specifically the quay cranes to serve each vessel. In addition, it calculates the number of internal trucks assigned to each quay crane, taking into consideration the limited availability of the internal trucks.

The model formulation is based on the following assumptions:

1. Each vessel has been previously planned for a berthing time, a promised departing time and a berthing location.
2. All quay cranes are identical and indexed sequentially according to increasing positions along the quay from left to right.
3. A Quay crane can't be included in the schedule until its required minimum number of internal trucks is available.
4. The handling operations of the vessel start only if its minimum number of quay cranes is available.

The following notations are used

Sets:

T : Set of time periods (indexed by *t*).

V : Set of vessels (indexed by *v*).

K : Set of quay cranes (indexed by *k*).

Parameters :

S_v : The berthing time of vessel *v*.

C_v : The promised departing time of vessel *v*.

N_v : The number of the loading and discharging containers of vessel *v* in containers assuming all containers are 20 foot containers.

l_v : The length of vessel *v*.

r_v : The berthing position of vessel *v*.

q_{max}^v : Maximum number of quay cranes that can be assigned simultaneously to vessel *v*.

q_{min}^v : Minimum number of quay cranes that can be assigned simultaneously to vessel *v*.

Q : Available total number of quay cranes.

I : Available total number of internal trucks.

i_{max} : Maximum number of internal trucks that can be assigned to each quay crane.

i_{min} : Minimum number of internal trucks that should be assigned d to each quay crane.

C_v^1 : Service cost rate given in units of US$1000 per hour for each vessel *v*.

C^2 : Operation cost rate of the quay crane given in units of US$1000 per quay crane-hour.

C^3 : Operation cost rate of the truck given in units of US$1000 per truck-hour.

μ : The average service time of the quay crane.

2τ : The average total travel time of the internal truck between the quay side and yard.

ɣ : The average service time of the yard crane.

M : A sufficiently large constant.

Decision variables

q_{vt}^k : *1*, if quay crane *k* is assigned to vessel *v* in time period *t*, *0* otherwise.

i_{vt}^k : Integer number to represent the number of internal trucks assigned to quay crane *k* when it works on vessel *v* in time period *t.*

x_{vt} : *1*, if vessel *v* is handled at time period *t*, *0* otherwise.

y_{vt} : *1*, if handling operation of vessel *v* is started at or before time period *t*, 0 otherwise.

z_{vt} : *1*, if handling operation of vessel *v* is not completed at time period *t*, 0 otherwise.

b_{kt} : Integer number to represent the position of quay crane *k* in time period *t*.

The model is formulated as follows:

Minimize Z = $C_v^1 \sum_{v\in V} (\max_{t=S_v,\ldots,C_v} (t \cdot x_{vt}) - S_v + 1) + C_2 \sum_{v\in V}\sum_{k\in K}\sum_{t\in T} q_{vt}^k + C_3 \sum_{v\in V}\sum_{k\in K}\sum_{t\in T} i_{vt}^k$

Subject to

$$\sum_{k\in K}\sum_{v\in V} i_{vt}^k \le I \qquad \forall t \in T \qquad (1)$$

$$\sum_{k\in K}\sum_{v\in V} q_{vt}^k \le Q \qquad \forall t \in T \qquad (2)$$

$$y_{vt} \le y_{v(t+1)} \qquad \forall v \in V, \forall t = S_v,\ldots,C_v \qquad (3)$$

$$z_{vt} \ge z_{v(t+1)} \qquad \forall v \in V, \forall t = S_v,\ldots,C_v \qquad (4)$$

$$z_{vt} + y_{vt} = x_{vt} + 1 \qquad \forall v \in V, \forall t = S_v,\ldots,C_v \qquad (5)$$

$$\frac{1}{\mu + 2\tau + \gamma} \sum_{k\in K}\sum_{t=S_v}^{C_v} i_{vt}^k \ge N_v \qquad \forall v \in V \qquad (6)$$

$$\sum_{k\in K}\sum_{t=1}^{S_v-1} q_{vt}^k = 0 \qquad \forall v \in V \qquad (7)$$

$$\sum_{k\in K}\sum_{t=C_v+1}^{T} q_{vt}^k = 0 \qquad \forall v \in V \qquad (8)$$

$$i_{vt}^k \le i_{\max} \cdot q_{vt}^k \qquad \forall v \in V, t \in T, k \in K \qquad (9)$$

$$i_{vt}^{k} \geq i_{\min} \cdot q_{vt}^{k} \qquad \forall v \in V, t \in T, k \in K \qquad (10)$$

$$\sum_{v \in V} q_{vt}^{k} \leq 1 \qquad \forall k \in K, t \in T \qquad (11)$$

$$\sum_{k \in K} q_{vt}^{k} \leq q_{\max}^{v} \cdot x_{vt} \qquad \forall v \in V, \forall t = S_v, ..., C_v \qquad (12)$$

$$\sum_{k \in K} q_{vt}^{k} \geq q_{\min}^{v} \cdot x_{vt} \qquad \forall v \in V, \forall t = S_v, ..., C_v \qquad (13)$$

$$b_{kt} \leq b_{(k+1)t} \qquad \forall k = 1, ..., K-1, t \in T \qquad (14)$$

$$b_{kt} \geq r_v \cdot q_{vt}^{k} \qquad \forall v \in V, \forall k \in K, t = S_v, ..., C_v \qquad (15)$$

$$b_{kt} \leq (r_v + l_v - 1) q_{vt}^{k} + M(1 - q_{vt}^{k}) \qquad \forall v \in V, \forall k \in K, \forall t = S_v, ..., C_v \qquad (16)$$

$$\max_{k \in \{K | q_{vt}^{k} = 1\}} (k \cdot q_{vt}^{k}) - \min_{k \in \{K | q_{vt}^{k} = 1\}} (k \cdot q_{vt}^{k}) + 1 <= \sum_{k \in K} q_{vt}^{k}$$
$$\forall v \in V, \forall k \in K, \forall t = S_v, ..., C_v \qquad (17)$$

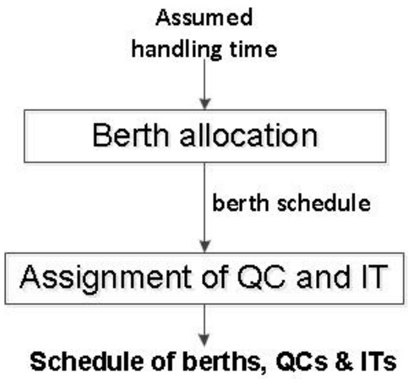
$$x_{vt}, y_{vt}, z_{vt}, q_{vt}^{k} \in \{0,1\} \qquad (18)$$

$$f_{vt}, b_{kt}, i_{vt}^{k} \geq 0 \ \ and\ Integer \qquad (19)$$

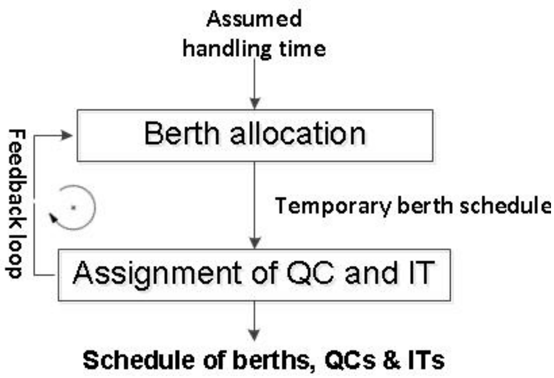
We model the tradeoff between the service quality and the operation cost by a weighted objective function which aims at minimizing the total service cost of all the vessels. The first term represents the service quality cost, represented in term of the handling time of the vessel which is calculated as the difference between the completion time and the berthing time plus one. Note that $Max_{t=S_v,...C_v}(t \cdot x_{vt})$ is the completion time of the vessel v .The second and third terms represent the operation cost of the quay cranes and internal trucks utilized in the handling plan respectively. Constraints (1) and (2) ensure that at each time period, the number of quay cranes and internal trucks assigned to all vessels do not exceed the available numbers. Constraint (3) allows the handling operations of the vessel to start at any time period between $S_v$and C_v. Constraint (4) allows the handling operations to be completed at any time period between $S_v$and C_v. Constraint (5) guarantees that once the handling operations are started, the operations must be performed without interruption until the handling operations are completed. Constraint (6) ensures that the containers on each vessel must be completely handled within its berthing stay. Constraints (7) and (8) ensure that no internal trucks or quay cranes are assigned to any vessel before its berthing time or after its departure time. Constraints (9) and (10) specify that the number of internal trucks assigned to each quay crane is between the minimum and maximum allowed values. Constraint (11) guarantees that any quay crane can be assigned to one and only one vessel at each time period. Constraint (12) and (13) ensure that the number of quay cranes assigned to each vessel is between the maximum and minimum specified limits. Constraint (14) ensures that the positions of the quay cranes along the quay are respected by the index of the quay cranes. This means that the position of the quay crane *k* is always lower than the position of the quay crane (*k+1)*. Constraints (15) and (16) ensure that if quay crane *k* is assigned to vessel *v*, then the position of the quay crane *k* must be within the berthing positions covered by vessel *v*. Constraint (17) ensures that the quay cranes between the most left and the right most quay cranes, which are assigned to the vessel, are all employed to serve the vessel. Constraints (18) and (19) are binary and integer constraints respectively. The proposed model can be represented as an MIP mode, as shown in the appendix, to be solved by any MIP solver.

Allocating berths, quay cranes and internal trucks to arriving vessels

The allocation of berths, quay cranes and internal trucks can be solved in different manners. According to Bierwirth and Meisel (2010), three approaches can be used, namely sequential approach, feedback loop approach and simultaneous approach. Figure 2 shows the concept of the sequential and feedback loop approaches. The simultaneous approach is realized by merging the two models into one model. Table 2 shows the advantages and disadvantages of the different approaches.



(a)



(b)

Figure 2 : sequential approach (a) and feedback loop approach (b).

Table 1: comparison of planning approaches

Approaches	Advantages	Disadvantages
1) Sequential	• Consider performance of sub-problems	• Ignore interactions of sub-problems
2) feedback loop	• Consider interactions and performance of sub-problems	• Depends on assumed variables • Need experience with value of variables • Non- convergence to stable state

3) Simultaneous	• Consider interactions of sub-problems	• More complex • Sometimes, unsolvable • Not ensure the performance of • all sub-problems

In the next section, we present a model for allocating berths, quay cranes and internal trucks simultaneously to incoming vessels.

An integrated berth, quay crane and internal truck allocation

The continuous berth layout, limited time-variant quay crane assignments and dynamic arrival of vessels are considered in the proposed model.

The model is formulated based on the following assumptions:
1. Once, the vessel is berthed, its service starts and can't be interrupted until it is completed.
2. The planning horizon is divided into equal-sized time periods, each of I hour.
3. The quay is divided into equal-sized berthing positions, each of 50 m.
4. There are no physical or technical restrictions such as vessel draft and water depth.
5. A minimum and a maximum number of quay cranes are determined for each vessel and vessel can't be berthed until its minimum number of quay cranes is made available to serve it.
6. Each of the quay cranes assigned to a vessel, at the same time period, is assigned the same number of internal trucks. This assumption may not hold in practice but it is made to simplify the solution process.
7. The change in the number of quay cranes assigned to a vessel during its service is limited to one change at most, which is referred to as limited time-variant quay crane assignments and are used in (Giallombardo et al., 2010; Zhang et al., 2010). Referring to vessel B in figure 1, we can note that its assigned numbers of quay cranes change only one time during its service at time period 5.
8. A minimum and a maximum number of internal trucks are determined for each quay crane such that that the idle times of the cranes and internal trucks are practically acceptable. It should be noted that assigning too much trucks to the quay crane, increases the waiting times of internal trucks as well as traffic congestion at the quay side.

Notations

SETs

U : set of vessels to be served, K= {1,2,...,v} (indexed by k).
B : set of berthing positions, B= {1,2,...,m} (indexed by i).
Y : set of time periods, Y= {1,2,...,n} (indexed by j)

Parameters:

l_k : The length of vessel *k*, including the allowance distance between vessels**.**
$\boldsymbol{a_k}$ **:** The expected arrival time of vessel *k*.
d_k : The expected departure time of vessel *k*.
$\overline{b_k}$: The desired berthing position of vessel *k*.
m : Total number of berthing positions.
n : Total number of time periods.
Q : Total number of available quay cranes.
T : Total number of available internal trucks.
N_k : The number of the loading and discharging containers of vessel *k* in (TEU), **assuming all containers are 20 foot containers**
r_k^{max} : Maximum number of quay cranes that can be assigned simultaneously to vessel *k*.
r_k^{min} : Minimum number of quay cranes that can be assigned simultaneously to vessel *k*.
t_{max} : Maximum number of internal trucks that can be assigned to each quay crane.
t_{min} : Minimum number of internal trucks that can be assigned to each quay crane.
R : Feasible range of internal trucks that can be assigned to each quay crane, $R = [t_{min}, t_{max}]$.
p : Average productivity of each QC per one IT (TEU/IT) per hour.
c_1 : The penalty cost for unit distance deviation between B_k and $\overline{b_k}$ in units of \$1000.
c_2 : The penalty cost for the delayed berthing time of the vessel in units of \$1000 per hour.
c_3 : The penalty cost for the delayed departure time of the vessel in units of \$1000 per hour.
M : A sufficiently large constant.

Decision variables

B_k : Integer, the berthing position of vessel *k*.
T_k : Integer, the berthing time of vessel *k*.
C_k : Integer, the completion time of vessel *k*.
x_{ijk} : 1, if the berthing position *i* is occupied by vessel *k* at time period *j*, 0 **otherwise.**
z_{ijk} : 1, if the first berthing position (B_k) and the berthing time (T_k) of vessel *k* are *i* and *j* respectively, 0 otherwise
q_{jk}^{c} : Integer, number of quay cranes assigned to vessel k at time period j if c internal trucks are allocated to each quay crane, $c \in R$.
S_{jk} : Integer, number of internal trucks allocated to serve vessel *k* at time period *j*.
h_{jk} : 1, if the vessel *k*is processed in time period *j* by at least one quay crane, 0 otherwise.
g_{jk} : 1, if there is a change in the number of the quay cranes assigned to vessel *k* at time period *j*, 0 otherwise.

The mathematical model

$$\text{Minimize } \sum_{k=1}^{v}\left(c_1 \,|\, B_k - \bar{b}_k \,|\, + c_2 (T_k - a_k)^+ + c_3 (C_k - d_k)^+\right)$$

Subject to

$$\sum_{k=1}^{v} x_{ijk} \le 1 \qquad \forall i \in B, j \in Y \qquad (1)$$

$$\sum_{k=1}^{v} \sum_{c=t_{\min}}^{t_{\max}} q_{jk}^{c} \le Q \qquad \forall j \in Y \qquad (2)$$

$$\sum_{k=1}^{v} s_{jk} \le T \qquad \forall j \in Y \qquad (3)$$

$$p \sum_{j=1}^{n} \sum_{c=t_{\min}}^{t_{\max}} c \cdot q_{jk}^{c} \ge N_k \qquad \forall k \in U \qquad (4)$$

$$\sum_{j=1}^{n} h_{jk} = C_k - B_k \qquad \forall k \in U \qquad (5)$$

$$(j+1) h_{jk} \le C_k \qquad \forall j \in Y, k \in U \qquad (6)$$

$$j \cdot h_{jk} + M(1 - h_{jk}) \ge T_k \qquad \forall j \in Y, \forall k \in U \qquad (7)$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} z_{ijk} = 1 \qquad \forall k \in U \qquad (8)$$

$$\sum_{i=1}^{i'-1} \sum_{j=1}^{n} x_{ijk} + \sum_{i=i'+l_k}^{m} \sum_{j=1}^{n} x_{ijk} \le M\left(1 - \sum_{j=1}^{n} z_{i'jk}\right)$$

$$\forall i' = 2, \ldots, m - l_k, \forall k \in U \qquad (9)$$

$$\sum_{i=1+l_k}^{m} \sum_{j=1}^{n} x_{ijk} \le M\left(1 - \sum_{j=1}^{n} z_{1jk}\right) \qquad \forall k \in U \qquad (10)$$

$$\sum_{i=1}^{m} x_{ij} \le M \cdot h_{jk} \qquad \forall j \in Y, \forall k \in U \qquad (11)$$

$$l_k - \sum_{i=1}^{m} x_{ij} \le M\left(1 - h_{jk}\right) \qquad \forall j \in Y, \forall k \in U \qquad (12)$$

$$\sum_{c=t_{\min}}^{t_{\max}} q_{jk}^{c} \le r_k^{\max} \cdot h_{jk} \qquad \forall j \in Y, \forall k \in U \qquad (13)$$

$$\sum_{c=t_{\min}}^{t_{\max}} q_{jk}^{c} \ge r_k^{\min} \cdot h_{jk} \qquad \forall j \in Y, \forall k \in U \qquad (14)$$

$$T_k = \sum_{i=1}^{m} \sum_{j=1}^{n} j \cdot z_{ijk} \qquad \forall k \in U \qquad (15)$$

$$B_k = \sum_{i=1}^{m}\sum_{j=1}^{n} i \cdot z_{ijk} \qquad \forall k \in U \qquad (16)$$

$$s_{jk} = \sum_{c=t_{\min}}^{t_{\max}} c \cdot q_{jk}^{c} \qquad \forall k \in U, \forall j \in Y \qquad (17)$$

$$\left| \sum_{c=t_{\min}}^{t_{\max}} q_{jk}^{c} - \sum_{c=t_{\min}}^{t_{\max}} q_{(j-1)k}^{c} \right| + M \cdot (1 - g_{jk}) \geq M \cdot \left| h_{jk} - h_{(j-1)k} \right| \qquad \forall j = 2,\ldots,n, k \in U \qquad (18)$$

$$\left| \sum_{c=t_{\min}}^{t_{\max}} q_{jk}^{c} - \sum_{c=t_{\min}}^{t_{\max}} q_{(j-1)k}^{c} \right| + M \cdot g_{jk} \leq M \cdot \left| h_{jk} - h_{(j-1)k} \right| \qquad \forall j = 2,\ldots,n, k \in U \qquad (19)$$

$$\sum_{j=1}^{n} g_{jk} \leq 1 \qquad \forall k \in U \qquad (20)$$

$$C_k \leq n \ , \ B_k \leq m - l_k + 1 \text{ and } B_k \geq a_k \qquad \forall k \in U \qquad (21)$$

$$x_{ijk}, z_{ijk}, h_{jk}, g_{jk} \in \{0,1\} \qquad (22)$$

The objective function aims at minimizing the weighted sum of the handling costs of containers. The handling cost of containers comprises three cost components. The first cost is the cost of deviation between the berthing position of the vessel and the yard where the outbound containers of the corresponding vessel are stored. The yard is represented by the desired berthing position of the vessel. The second cost is the penalty cost when the berthing time of vessel is latter than its expected arrival time. The third cost is the penalty cost incurred when the departure time of the vessel is latter than its expected departure time. Constraint (1) ensures that each berthing position is occupied by no more than one vessel at each time period. Constraints (2) and (3) ensure that at each time period, the number of quay cranes and internal trucks utilized do not exceed the total available number of quay cranes and internal trucks respectively. Constraint (4) ensures that all the containers on each vessel must be completely handled. It should be noted that this constraint is formulated as in the work of Karam et al. (2015) in which they estimated the handling rate of the quay crane as the product of the number of internal trucks assigned to the quay crane and the average productivity of the quay crane per internal truck (TEU/IT per hour). They also assumed that the quay cranes are assigned to non-overlapping segments of the vessel. Thus, there is no interference among the assigned quay cranes and so, the handling rate of the vessel is the sum of the handling rates of all quay cranes assigned to the vessel. It is also important to note that the number of internal trucks c assigned to each quay crane is not a decision variable and is taken from the range R. It was possible to insert a decision variable representing the number of internal trucks assigned to each quay crane and also insert more constraints to specify the upper and lower limits for this number. But this way the complexity of the formulation will increase. Constraint (5) defines the variable h_{jk}. Also, constraint (5) guarantees that vessel operation can't be interrupted once it starts. Constraints (6) and (7) define the completion time and the berthing time for each vessel respectively. Constraint (8) guarantees that at all timer periods of the planning horizon, no more than one berthing position and one time period can be set as the berthing position (B_k) and the berthing time (T_k) for each vessel respectively. Constraints (9)-(12) ensure that for each vessel, x_{ijk}=1 only at the grids covered by the rectangle of vessel k in the time-space diagram. These grids belong to [$T_k,\ldots,C_k$] and [$B_k,\ldots,B_k+l_k-1$]. Constraints (13)-(14) ensure that the number of quay cranes assigned to each vessel is between the maximum and minimum specified limits. Constraints (15)-(16) define the variable z_{ijk}. Constraint (17) defines the variable s_{jk} .constraints (18)-(20) guarantee that the change in the number of quay cranes assigned to a vessel doesn't occur more than one time during the vessel service. It can be noted that the change in the number of quay cranes assigned to the vessel can be easily adjusted by constraint (20). For example, if the port planner prefers to set the number of crane changes to two times the right hand side of constraint (20) is set to two instead of one. Constraints (21) and (22) are the non-negativity and binary constraints respectively.

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